Radiant wall panel heating with mechanical ventilation

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Summary This paper outlines the search for a design method for radiant wall panel heating using steel pipe coils embedded in concrete cross walls, in a local authority housing scheme (500 dwellings). Using an electrical analogue method, it provides details of heat outputs to be expected for variable pipe spacings and temperatures, for a fixed wall thickness or pipe depth. Anticipated room conditions and control problems are investigated. The radiant heating is linked with a mechanical ventilation system in each dwelling to aid control and minimise the risk of condensation. A cost comparison with radiators is given (1970 prices) and some details of site tests.

Radiant wall panel heating with mechanical ventilation

MAX FORDHAM, MA, FCIBS, and HELENE RYDING, BSc, MCIBS

1 Introduction

Construction of this system of heating, employed in a local authority housing scheme at Alexandra Road, London NW8, has been described in an earlier paper[1]. The current paper looks at the theory and practice of the scheme as designed and built.

The housing is heated by steel pipe coils cast into 178mm concrete tenancy separating walls, and fed by water from the district boiler-house. As far as the authors can tell, this method of heating has not been used before on such a scale, if at all. It was devised because of the particular requirements of the building and also because of its cost advantages over equivalent radiator schemes.

The acoustic precautions for the housing block nearest the railway necessitated the provision of mechanical ventilation, and it was found possible to extend this to the other blocks. Each dwelling therefore has provision for mechanical ventilation at the rate of 1½ air-changes per hour. The mechanical ventilation, combined with a small heater battery which can be bypassed, provides the means of control for the tenant, since the wall coils are centrally controlled at the boiler-house. By this method of constant background heating from the coils, plus mechanical ventilation with controllable heating or cooling, the risk of condensation should be eliminated.

1.1 Radiant panels

Although the design procedure for floor and ceiling panel heating is well established, there is little published information on wall panel heating. The CIBS Guide[2] Table B 1.14 gives outputs for heated walls for wall surface temperatures of 25-35°C (41-140W/m²), but does not indicate how these temperatures are to be achieved.

Investigations were therefore begun from first principles.

Calculation of theoretical output

List of symbols used in Section 2

c_p	specific heat at constant pressure	J/kg K
d	depth of buried pipe (to centre line)	m
e	spacing of pipes	m
g	acceleration due to gravity	9.81m/s²
h	coefficient of convective heat transfer	W/m² K
$\bar{h}$	mean ,, ,, ,,	W/m² K
k	thermal conductivity	W/m K
k_c	,, ,, of concrete	,,
k_f	,, ,, of fluid	,,
k_p	,, ,, of plaster	,,
ℓ	height of plate	m
n	constant	
q_c	heat output by conduction	W
q_{con}	,, ,, by convection at surface	W/m²
q_r	,, ,, by radiation at surface	W/m²
q_{xc}	heat flow in x direction in concrete	W
q_{xp}	,, ,, ,, in plaster	,,
q_{yc}	,, ,, in y direction in concrete	,,
q_{yp}	,, ,, ,, in plaster	,,
r	radius of pipe	m
t	temperature in wall or fluid	°C
t_{ai}	temperature of air in room	,,
t_c	temperature at surface of concrete	,,
t_o	temperature of pipe	,,
t_p	temperature at surface of plaster	,,
t_s	temperature at surface of wall	,,
x_c	half concrete wall thickness	m
x_p	plaster thickness	m
C	constant	
A	cross-sectional area of wall	m²
Gr	Grashof Number $\frac{g\beta\ell^3\rho^2\Delta t}{\mu^2}$	dimensionless
Nu	Nusselt Number $\frac{h\ell}{k}$	,,
Pr	Prandtl Number $\frac{\mu c_p}{k}$	,,
T_{rm}	absolute mean radiant temp. of surfaces seen by wall	K
T_s	absolute temperature of wall surface	,,
β	coefficient of volumetric expansion of fluid	/K
ϵ	emissivity of surface	dimensionless
μ	kinematic viscosity of fluid	kg/ms
ρ	density of fluid	kg/m³
σ	Stefan-Boltzmann constant	5.67 x 10⁻⁸
$\Delta t = t_s - t_{ai}$	temperature difference between plate surface and fluid	°C

2.1 Heat output by conduction of pipe embedded in concrete

The situation under investigation is shown in Fig. 1. Taking the pipe as a point source, and considering only one pipe, for unit length of wall, the heat output is

In concrete: $q_{xc} = -k_c A \frac{\partial t}{\partial x} \quad x, y = o \quad t = t_o$

$$q_{yc} = -k_c A \frac{\partial t}{\partial y} \quad x = x_c \quad t = t_c$$

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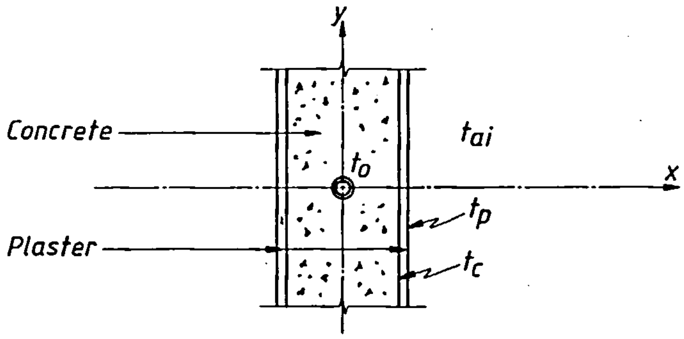


Fig. 1. Conduction in wall.

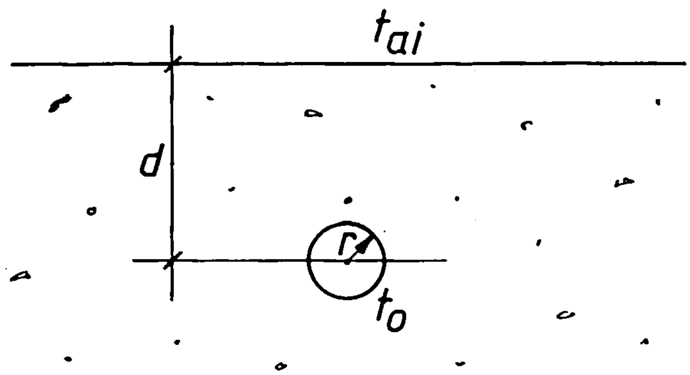


Fig. 2. Approximate solution for pipe buried in a solid.

In plaster: $q_{xp} = -k_p A \dfrac{\partial t}{\partial x}$ $\quad x = x_c \quad t = t_c$

$q_{yc} = -k_c A \dfrac{\partial t}{\partial y}$ $\quad x = x_c + x_p$

$q_{xc} = q_{xp}$ $\quad k_p \dfrac{(\partial t)}{(\partial x)}_{x_c + x_p} = h(t_p - t_{ai})$

$q_{yc} = q_{yp}$

As a simplification, replace plaster by extra concrete.
i.e. $x'_c = x_c + x_p k_c / k_p$ $\quad q_{xc} = -k_c A \dfrac{\partial t}{\partial x}$

$q_{yc} = -k_c A \dfrac{\partial t}{\partial y}$

This equation is difficult to solve, especially as no computer was available at the time, since the equation is cylindrical and the boundary conditions rectangular.

2.1.2 Approximate solutions

For a pipe buried in a solid [3]
For unit length of pipe:

$$q_c = \frac{2\pi k \quad (t_o - t_s)}{\log\left[(d/r) + \sqrt{\{(d/r)^2 - 1\}}\right]}$$

$k_c = 1.4\ W/m^2 K$ $d = x_c + \dfrac{x_p K_c}{K_p} = 136mm$
$k_p = 0.37 W/m^2 K$
$x_c = 89mm$
$x_p = 12.5mm$
t_o = pipe temperature
$r = 10mm$

$$q_c = \frac{2\pi\, 1.4 \quad (74 - t_s)}{\log\left[(136/10) + \sqrt{\{(136/10)^2 - 1\}}\right]} = 2.66\,(74 - t_s)\ \text{W/m run}$$

For a row of pipes buried in a solid **[4] (spacing e = 300mm)**
For any one pipe, in the midplane of a solid, and unit length of pipe:

$$q_c = \frac{2\pi k \quad (t_o - t_s)}{\log\left[(e/\pi r)\ \{\sinh(\pi d/e)\}\right]} = \frac{2\pi k}{\log\left[300/10\pi \sinh \pi 136/300\right]}$$

Overall heat output in $W/m^2 = q/e = q/0.3 = 10.00\,(t_o - t_s)$

But this is for both faces, so heat output per wall surface $= 5.00\,(t_o - t_s)$

2.2 Heat transfer by convection at wall surface

A general expression for the heat transfer by natural convection is given by:
$q_{con} = hA\,(t_s - t_{ai}) = hA\Delta t$
where h is given by:
$Nu = C(Gr.Pr)^n$
Using values for air at 300K[5]
$k_f = 0.02624\ W/m\ K$
$\beta = 3.33 \times 10^{-3}/K$
$\rho = 1.1774\ kg/m^3$
$\mu = 1.983 \times 10^{-5}\ kg/ms$
$c_p = 1006\ J/kg\ K$
$Pr = 0.708$
This gives $Gr = 11.5 \times 10^7 \ell^3 \Delta t$
$Nu = 0.38 \times 10^2 h\ell$
and $Gr.Pr = 8.142 \times 10^7 \ell^3 \Delta t$

For Δt in the range $10°C < \Delta t < 30°C$
ℓ in the range $0.5m < \ell < 3.5m$
$Gr.Pr$ lies in the range $1.018 \times 10^8 < Gr.Pr < 7.694 \times 10^{12}$

The transition from laminar to turbulent flow occurs in the range
$10^4 < Gr.Pr < 10^9$ laminar
$Gr.Pr > 10^9$ turbulent

Table 1. Heat transfer by convection at wall surface.

Δt K	ℓ m	$Gr.Pr$	Flow	$\overline{Nu}$	$\overline{h}$ W/m²K
10	0.5	1.018×10^8	laminar	51.83	2.72
	1	8.142×10^8	,,	87.16	2.18
	2	6.514×10^9	turbulent	177.40	2.32
	2.5	1.272×10^{10}	,,	231.85	2.43
15	0.5	1.527×10^8	laminar	57.36	3.010
	1	1.22×10^9	turbulent	90.77	2.38
	2	9.77×10^9	,,	208.63	2.73
	2.5	1.908×10^{10}	,,	272.67	2.86
20	0.5	2.036×10^8	laminar	61.64	3.23
	1	1.628×10^9	turbulent	101.88	2.67
	2	1.302×10^{10}	,,	234.02	3.07
	2.5	2.544×10^{10}	,,	305.93	3.21

For convection at the surface of a heated vertical plate

$\overline{Nu} = C\,(Gr.Pr)^n K$ [6]

and $q_{con} = h A \Delta t$

where Nu is the overall Nusselt number $= \dfrac{\bar{h}l}{k}$

$\bar{h}$ is the overall heat transfer coefficient

$\bar{h} = \dfrac{q_{con}}{A\Delta t}$

and K is a constant

For laminar flow[7] $C = 0.8$ $n = 0.25$

$$K = \left[1 + \left(\frac{(1+1)}{\sqrt{Pr}}\right)^2\right]^{-1/4} = 0.645$$

turbulent flow $C = 0.0246$ $n = 0.4$

$$K = \left[\frac{Pr^{1/6}}{1 + 0.494\,Pr^{2/3}}\right]^{2/5} = 0.856$$

This gives values for $\bar{h}$ as shown in Table 1.

These results were compared with Section C3 of the IHVE Guide [2], heat emission for plane surfaces by natural convection, using the figures for vertical surfaces, based on $h = 1.9\,(\Delta t)^{0.25}$. This formula appears to be based on McAdams' basic equation for laminar flow for short (½in — 12in) vertical plates in the laminar range

$Nu = 0.59\,(Gr.Pr)^{0.25}$

whereas for a considerable part of the range for length of wall surfaces and temperature differences under consideration the range of $(Gr.Pr)$ lies in the turbulent range. McAdams gives for this range

$Nu = 0.13\,(Gr.Pr)^{1/3}$

which is independent of the length of vertical plate, giving $h = C(\Delta t)^{1/3}$, the formula commonly used for radiator heat output. Although the IHVE Guide states that the equation relating h_c to $(\Delta t)^{0.25}$ does not apply to freely exposed plane surfaces (i.e. whole walls), it does not give any guidance as to what range of surfaces it does apply to. For temperature differences of 10°C and 20°C, the IHVE Guide predicts $\bar{h} = 3.37$ and $4.00 W/m^2$ °C respectively.

However, checking further with the IHVE Guide, h_{so} (the combined convection and radiation surface heat transfer coefficient) ranges from 8-10W/m^2 °C (Table C3.9). Convection and radiation are usually in the proportions of 35 and 65 per cent respectively, (from observation) giving $\bar{h}_c$ from 2.8-3.5W/m^2 °C which agrees well with IHVE Guide equation A3.5, where h_c for internal surface resistances is given as 3.0. It seems likely therefore, that for larger surfaces, Table C3.8 is over-optimistic in its prediction of heat output.

2.3 Heat transfer by radiation at wall surface

Heat output is given by the Stefan-Boltzmann law:

$q_r = \sigma\epsilon\,(t^4_s - t^4_{m})$

For a wall surface $\epsilon = 0.9$

therefore

$q_r = 5.67 \times 10^{-8} \times 0.9\,[(273 + t_s)^4 - (273 + t_m)^4]$

2.4 Heat output of walls

We are therefore in a position to make some estimates of the heat output of the walls for the particular spacing we have chosen.

By conduction

For pipe temperature of 74°C, and pipe spacing of 300mm

$q_c = 5(74 - t_s)\ W/m^2$

For $t_s = 30$°C $q_c = 220 W/m^2$

35°C $q_c = 195 W/m^2$

40°C $q_c = 170 W/m^2$

By convection

For 2.5m high wall neglecting edge effects, a room air temperature of 20°C.

$q_{con} = h\Delta t\ W/m^2$

$\Delta t = 10$°C ($t_s = 30$°C) $h_c = 2.43$ $q_{con} = 24 W/m^2$

15°C ($t_s = 35$°C) $= 2.86$ $q_{con} = 43 W/m^2$

20°C ($t_s = 40$°C) $= 3.21$ $q_{con} = 64 W/m^2$

By radiation

$q_r = 5.67 \times 10^{-8} \times 0.9\,[(273 + t_s)^4 - (273 + t_m)^4]$

taking $t_m = 20$°C

For $t_s = 30$°C $q_r = 54 W/m^2$

35°C $q_r = 83 W/m^2$

40°C $q_r = 114 W/m^2$

Table 2 tabulates these results.

2.5 Other spacings, pipe sizes and wall thicknesses

For other spacings, pipe sizes, and wall thicknesses, an estimate of the heat output could be obtained in a similar way, using the equations:

$q_c = q_{con} + q_r$

$$q_c = \frac{2\pi 1.4}{\log_e \pi l r \sinh \pi d/e}\,(t_o - t_s)$$

$q_{con} = \bar{h}A\,(t_s - t_{ai})$

$\dfrac{\bar{h}l}{k} = 0.0246\,[8.142 \times 10^7 l^3\,(t_s - t_{ai})]^{0.4} \times 0.856$

$q_r = 5.67 \times 10^{-8} \times 0.9\,[(273 + t_s)^4 - (273 + t_m)^4]$

3 Electrical analogue

An electrical analogue to the heating scheme was set up in the following way (Fig. 3). The area of the wall was represented by a sheet of resistance paper, on which were painted, using conducting paint, circles representing the pipes and lines representing the constant ambient temperature of the room. The surface resistance at the wall was taken into account by extending the thickness of concrete by an amount whose resistance was equal to the surface resistance. A constant potential was applied between the pipes and the boundary line, and the corresponding equipotentials plotted using a voltmeter.

Dimensions are given in imperial units (with metric equivalents in brackets), as this was how the work was done.

Pipe spacings of 9, 14, 24in and ∞(229, 356, 610mm and ∞) were investigated for ¾in (19mm) o.d. pipes in the centre of 7in (178mm) concrete walls.

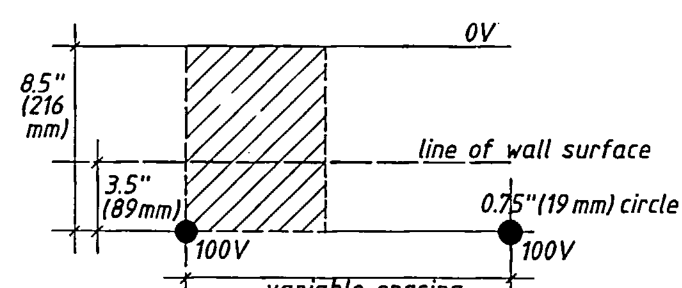


Fig. 3. Analogue diagram.

Table 2. Heat output of walls.

Surface temperature °C	Conduction W/m²	Convection W/m²	Radiation W/m²	Conv.+Rad. W/m²
30	220	24	54	78
35	195	43	83	126
40	170	64	114	178

Surface conductance (radiation and natural convection)
concrete to air = 2 $Btu/ft^2\,°F\,h$
= (11.356 W/m^2K)
Thermal conductivity of concrete = 10 $Btu\,in/ft^2\,°F\,h$
= (1.442 W/mK)

In order to take account of the surface conductance, the thickness of the concrete was increased by 5in (125mm) either side. No account was taken of plaster in the original calculations.
Room air temp = 65°F (18.3°C)
Mean water temp = 165°F (73.9°C)

Equipotential lines were plotted round the pipes and measurements made at ½in (12.5mm) intervals along the line of the wall. A mean wall surface temperature was calculated as a percentage as follows:

$$\frac{\text{temperature at wall} - \text{temperature of room}}{\text{temperature of pipe} - \text{temperature of room}} \times 100\%$$

and the total heat emitted by the wall was calculated, expressed as output per unit length of pipe and per unit area of wall, for each face of the wall. The results obtained were checked by an alternative flow path method, which gave confirmatory upper and lower bounds. Checks were also made for the linearity of the resistance paper and the voltmeter.

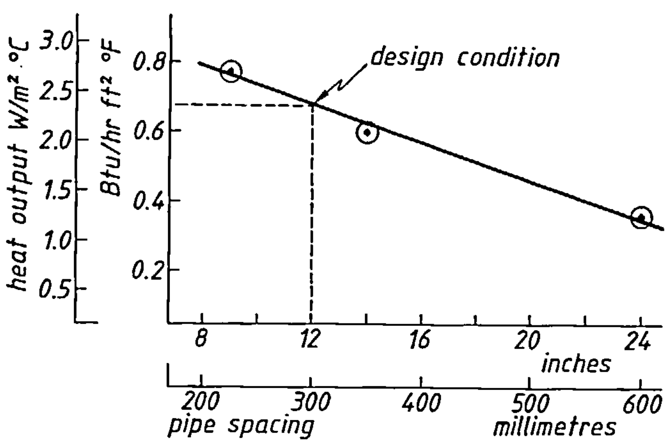


Fig. 4. Heat output per unit surface area for varying pipe spacing (one side of wall only).

Table 2. Electrical analogue results—Imperial units.

Pipe spacing	Mean wall surface temp	Actual mean wall surface temp	Heat flow per pipe	Heat flow per unit surface area
inches	per cent	°F	Btu/ft h°F	Btu ft² h°F
9	39.2	104.2	0.588	0.784
14	30.1	95.1	0.702	0.602
24	18.1	83.1	0.724	0.362

S.I. units

Pipe spacing	Mean wall surface temp	Actual mean wall surface temp	Heatflow per pipe	Heatflow per unit surface area
mm	per cent	°C	W/mK	W/m²K
229	39.2	40.1	1.02	4.45
356	30.1	35.1	1.22	3.42
610	18.1	38.4	1.25	2.05

The results in the analogue are shown in Figs. 4, 5 and 6. The graphs can also be used to predict the heat output for various pipe spacings and for various differences in temperature between the pipe and the room.

The design condition in Fig. 4 was taken to be 12in (300mm) spacing giving an output of 0.675$Btu/h\,ft^2$ °F (3.83W/m^2K). For a temperature difference of 100°F (55.6°C) between the pipe and room:
Output = 0.675 × 100 = 67.5$Btu/h\,ft^2$ (213W/m^2).

A comparison was made of results predicted by the graph and the results produced by J. Vidal [*] in his paper "Experimental calculation by electrical analogue of temperature distribution in radiant panels". This paper describes a more complicated electrical analogue but uses the same basic technique of applied voltage to points on resistance paper. The conditions investigated are for floor panels only, but some of the conditions can be compared, and give quite good agreement. (See Tables 3 and 4.)

Unfortunately there is no way of applying these results to walls of different thicknesses or pipes at varying depths, but it is hoped to investigate these in the future.

4 Installation design

Symbols used in Section 4

A_i, A_j, A_k	Area of wall i, j, k	m²
H_{BR}	Heat output of bedroom wall	W
H_L	Heat output of living room wall	W
Q_v	Ventilation heat loss per unit temp rise	W/K
Q_s	Structural heat loss per unit temp rise	W/K
T_1, T_2, T_3 etc	Temperature in room 1, 2, 3 etc.	°C
T_o	External temperature	°C
U_i, U_j, U_k	U value of wall i, j, k	W/m²K

4.1 Heat loss estimation (Figs. 7, 8 and 9)

A quick heat loss estimation using the conventional method of heat loss calculation for the dwelling was done. Room temperatures were those recommended by Parker Morris, plus additional requirements by the client. The original design had been done with living-room temperatures of 18°C, and this was not adjusted later, as it was expected that the increase of mean radiant temperature would give the required environmental temperatures. Air-change rates were taken as 1½ air-changes per hour throughout the dwelling. This air was allocated throughout the dwelling according to the positions of doors, windows and kitchen and lavatory extract fans, assuming the mechanical ventilation system was not working. This gave a total heat loss for the dwelling of about 3.5kW.

4.2 Check on available wall area

A check was made of the available wall area for heating, and at 24m², with a heat output of 5.1kW at 213W/m², one wall was adequate to heat the whole dwelling.

4.3 Heat balance

Heat losses were again calculated, but as a set of simultaneous equations, as follows. An equation balancing heat gains and losses for each room was set up, e.g.:

Living room (heated)

$$H_L = (Q_v + Q_s)(T_1 - T_0) + \Sigma A_i U_i (T_1 - T_2) + \Sigma A_j U_j (t_1 - T_3)$$

Kitchen (unheated)

$$0 = (Q_v + Q_s)(T_4 - T_0) + \Sigma A_i U_i (T_4 - T_2) + \Sigma A_j U_j (t_4 - T_3) + \Sigma A_k U_k (T_4 - T_5)$$

Taking the temperatures of the living room and bedroom as given at 18°C, the unknowns are T_2, T_4, T_5, and the heat outputs H_L and H_{BI}, i.e. five unknowns for which we have five equations.

The solution for this particular dwelling is given in Table 5, together with the client's required temperatures.

4.4 Coil design

The area of wall required for heating was checked against the wall area available. In the case of the bedroom this was ample, but in the living room the available area was only 9.7m² as against 12.2m² required. Ways of decreasing the heat loss of the living room were sought, but this was found difficult, as the main heat loss was from the full height full width single glazing. No money was available for double glazing, so that this heat loss could not be reduced. The roof, however, had insulation added, reducing the U value from 1.14W/m²K in the heat loss calculations to 0.82. In addition, it was felt that the sliding door between the bedroom and living room would not provide much insulation between the two rooms, and so the bookshelf/lobby space between the rooms could be considered to contribute to the living room heating, to make up the rest of the area. Coils were then designed to take up this area, including tails, as shown in Fig. 10.

4.5 Gable end condition

Coils occur on external walls only at gable ends where the construction is as shown in Fig. 11.

Thermal resistance from pipe coil to internal space = 0.21m²K/w
Thermal resistance from pipe coil to external air = 0.88m²K/w

$$\text{Therefore } \frac{\text{heat output internally}}{\text{heat output externally}} = \frac{0.88}{0.21} = 4.2$$

Coils on external walls have been designed in exactly the same way as other walls, for simplicity. Should overheating occur, then the amount of water circulating through the coils will be reduced.

Table 3. Vidal conditions.

Pipe depth in slab = 70mm
Surface resistance (upper surface) = 10m²h K/kcal = 11.63m²K/W
Conductivity of concrete = 1kcal/hmK = 1.163W/m²K
Pipe temp = 50°C
Room temp = 20°C

Pipe spacing mm	500	350	200
Heat output k cal/m²h	60.0	83.0	121.0
Heat output W/m²	69.8	96.5	140.7
Mean surface temp °C	26.0	28.3	32.1

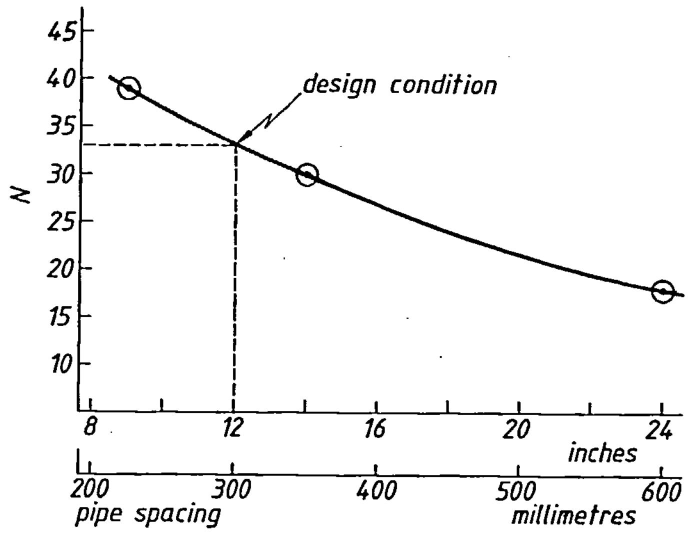
Table 4. Predictions from Graphs 1 and 2.

Based on : pipe depth in wall = 89mm (3.5in)
Surface resistance = 11.356m²K/W
Concrete conductivity = 1.442W/mK
Pipe temp = 50°C
Room temp = 20°C

Pipe spacing mm	500	350	200
Heat output W/m²	76.8	104	143
Mean surface temp °C	26.5	29.3	32.3

Table 5. Heat requirements for dwelling.

Room	Design temp °C	Heat balance temp °C	Heat required (+10 per cent)	Area of wall (based on 213W/m²)
1. Living Room	18	18 (given)	2.6kW	12.2
2. Entry	13	14	0	—
3. Bedroom	18	18 (given)	1.3kW	6.1
4. Kitchen	13	14	0	—
5. Bathroom	None (assumed 10°C)	14	0	—



$$N = \frac{\text{temp. on wall} - \text{temp. in room}}{\text{temp. of pipe} - \text{temp. in room}} \times 100$$

Fig. 5. Mean surface temperature for various spacings.

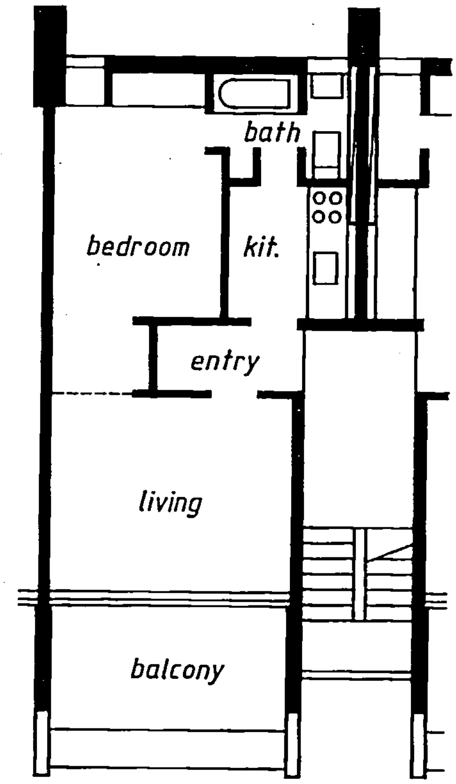


Fig. 7. Typical 2 person flat.

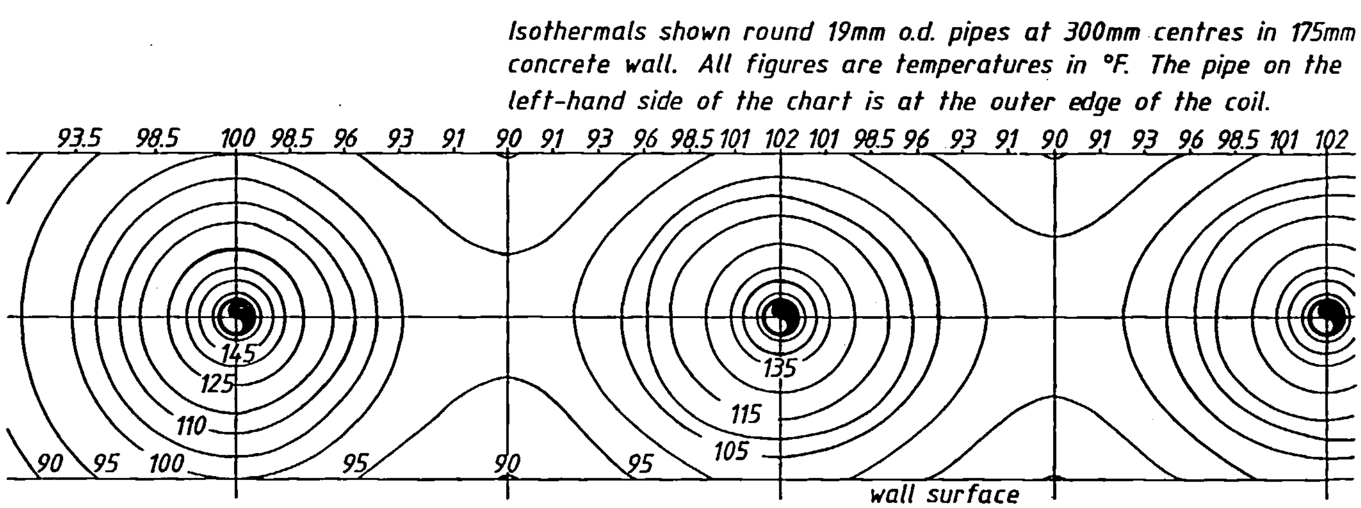


Fig. 6. Temperature distribution round pipe.

4.6 Fan convectors

These were designed in two sizes, for the different dwelling types. The fans were designed at 25 and 50 litres/s, to work against $125N/m^2$, and were 150mm centrifugal fans working at 2900 rev/min. The heater batteries were designed to heat the relevant amount of air from —1°C to 50°C when supplied with water at 82°C flow and 66°C return. This duty was chosen to provide some measure of heating to the dwelling, rather than just provide warmed air. The water temperatures for the heater battery are those for the coils, as the two systems are in series, the water passing through the heater battery first. This was largely because of the later addition on the fan convectors, which, if they had been on a constant temperature circuit would have necessitated extra pipework.

Each dwelling has two or three outlets from the fan convector to provide heating in rooms not on heated walls.

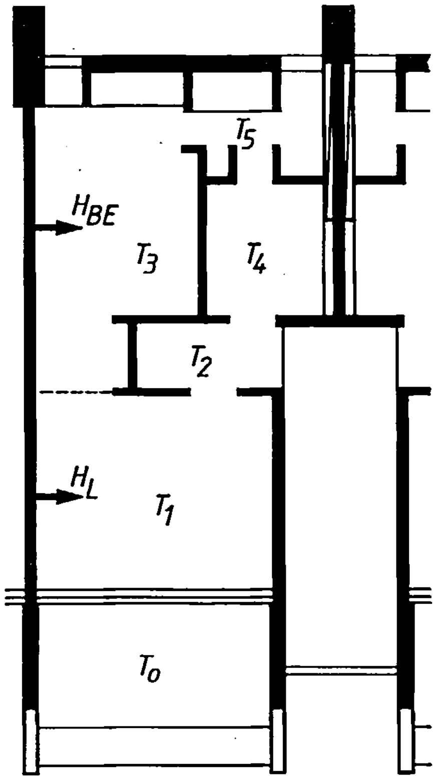
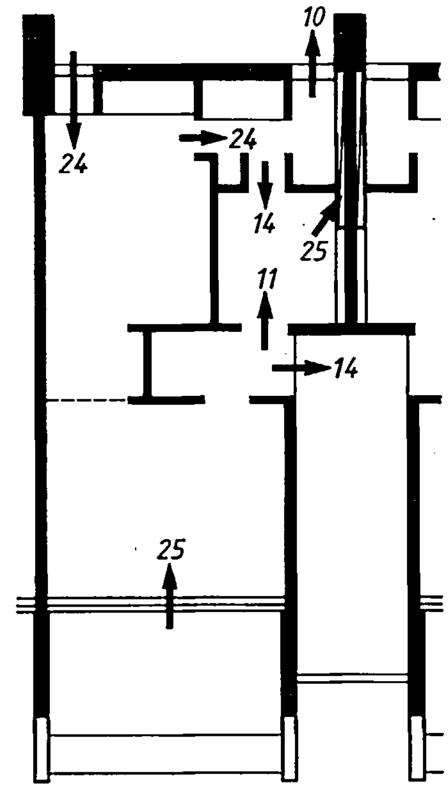


Fig. 8. Heating.



Figures are litres/second.

1.5 air changes/hour for whole dwelling = 49 l/s.

Fig. 9. Natural ventilation.

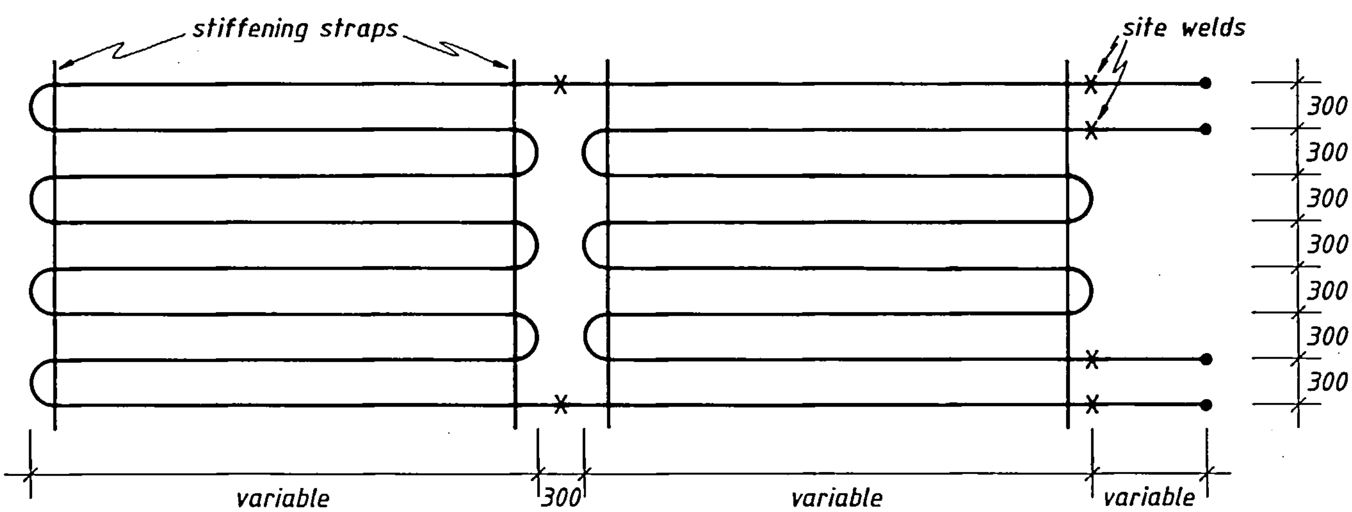


Fig. 10. Typical coil.

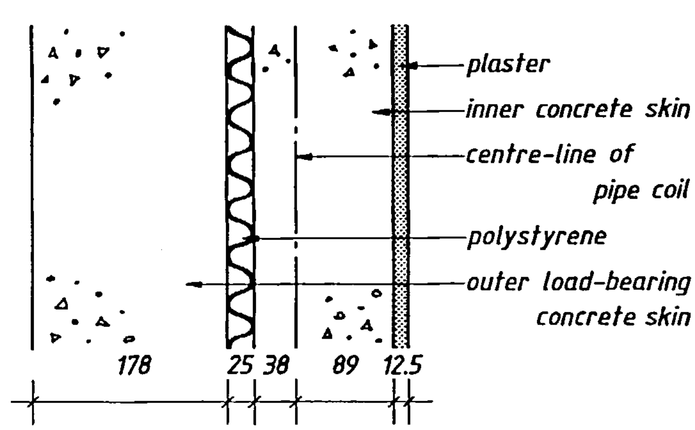


Fig. 11. Gable end section.

5 Comfort and control

Control of the heating is achieved in a number of ways. The basic level of heating provided for all tenants in one block is set by the water flow temperature, controlled in the boilerhouse by a weather-sensitive compensator. The high thermal capacity of the building does not permit sensitive control according to rapidly fluctuating weather conditions.

Weather data from the Meteorological Office was analysed to find the frequency and magnitude of rapid fluctuations. Checks were also made of the thermal capacity of the building, both for heating up and cooling down. Investigations were made as to the effect of small changes in outside temperature on inside temperatures. From these investigations, it would appear that the worst problems will be overheating in Spring, when an outside temperature rise of 5°C in two hours will produce an internal temperature rise of 3°C and the building will take 3.5-4 hours to cool down. This appears likely to happen 2-3 times a year. On these occasions, the tenant will be forced to cool the dwelling by opening the windows, to throw away the heat.

List of symbols used in Section 5

h	combined convection and radiation heat transfer coefficient	W/m²°C
q_w	heat output of wall	W
t_{ai}	internal air temperature	°C
t_{ao}	external air temperature	°C
t_{ei}	room environmental temperature	°C
t_f	flow water temperature	°C
t_p	mean water temperature	°C
t_r	return water temperature	°C
t_s	wall surface temperature	°C
t_{ri}	mean radiant temperature of room	°C
A	area of wall	m²
C_v	ventilation loss per unit temp rise	W/K
N	air change rate	/h
Q_f	heat loss through building fabric	W
Q_v	ventilation heat loss	W
U	U value of wall	W/m²K
V	volume of room	m³
W	rate of water flowing in pipes	m³/sec

5.1 Environmental temperature, theoretical

Living room
For radiant heating systems

$$t_{ei} - t_{ai} = \frac{Q_v}{4.8\Sigma A}$$

$$Q_v = C_v(t_{ei} - t_{ao})$$

$$\frac{1}{C_v} = \frac{1}{0.33NV} + \frac{1}{4.8\Sigma A}$$

$$4.8\Sigma A\,(t_{ei} - t_{ai}) = (t_{ei} - t_{ao})\ \frac{0.33NV \quad 4.8\Sigma A}{4.8\Sigma A \quad + 0.33NV}$$

For a living room with
$\Sigma A = 78.25\text{m}^2$ $V = 42.84\text{m}^3$
This reduces to:
$t_{ei} = (1 + 0.038N)\,t_{ai} - 0.038N\,t_{ao}$
For design conditions $t_{ei} = 21°C$ $N = 1.5/hr$ $t_{ao} = -1°C$
This gives room conditions of:
$t_{ei} = 21°C$ $t_{ai} = 19.8°C$ $t_{ri} = 21.6°C$

Alternatively

Heat provided by wall = 9.7 × 213 = 2000W
$\Sigma Q_f = \Sigma AU\,(t_{ei} - t_{ao})$ ΣAU external surfaces only

$$\frac{1}{C_v} = \frac{1}{0.33NV} + \frac{1}{4.8\Sigma A}$$

Given $\Sigma AU = 78\text{W/K}$ $t_{ao} = -1°C$
$\Sigma A = 78.25\text{m}^2$ $V = 42.84\text{m}^3$ $N = 1.5/\text{h}$.
This gives
$C_v = 20\text{W/K}$
and $Q_v = 20(t_{ei} - t_{ao})$
but $\Sigma Q_f + Q_v = 2000$
giving $t_{ei} = 19.4°C$
and $t_{ai} = 18.4°C$
$t_{ri} = 19.9°C$

Bedroom

As for Living room

$$(t_{ei} - t_{ai}) = (t_{ei} - t_{ao})\ \frac{0.33NV}{4.8\Sigma A + 0.33NV}$$

but $\Sigma A = 75.6\text{m}^2$ $V = 40.4\text{m}^3$
Hence $t_{ei} = t_{ai}(1 + 0.037N) - 0.037N\,t_{ao}$
For design conditions
$t_{ei} = 18.3°C$ $t_{ao} = -1°C$ $N = 1.5/\text{h}$
$t_{ai} = 17.3°C$ $t_{ri} = 18.8°C$

5.2 Weather data

The Meteorological Office was asked to supply details of temperature changes of 5°C or more over one hour and two hour periods. Details were provided for 1963 to 1970 inclusive, with dates and times. The figures were analysed as follows:—

Table 6. Changes in one hour.

	No of rises			No of falls		
Month	5°C	6°C	7°C	5°C	6°C	7°C
J						
F				1	1	
M			1			
A				2	1	
M		1		1		
J						
J						
A						
S						
O						
N	1					
D						

	Rise	Fall
Average change °C	6	5.3
No of occurrences over eight years	3	6

No of occurrences

Time of day	Rise	Fall
0800 — 1200	3	1
1200 — 1600		3
1600 — 2000		1
2000 — 2400		1

The conclusion drawn from Table 6 was that large changes in one hour were rare enough to be discounted.
Changes in two hours are analysed below in Table 7.
Some of these two-hour periods overlap, i.e. 0800-1000, and 0900-1100, on the same day. An analysis of the rises in the morning was made as shown in Table 8.
In all these cases, there is an overlapping period, so that the total number of rapid rises is artificially increased, and should be reduced from 96 to 76.

Table 7. Changes in two hours.

	No of rises			No of falls		
Month	5°C	6°C	7°C	5°C	6°C	7°C
J						
F		1				
M	11	1				
A	14	1		1		
M	13	4		3		
J	17	1		2		
J	6	2		1	1	
A	4	2				
S	8	1				
O	7	1				
N	1			2		
D		1		1		

	Rise	Fall
Average change °C	5.16	5.09
No of occurrences over eight years	96	11

No of occurrences

Time of day	Rise	Fall
0400—0800	25	2
0800—1200	66	1
1200—1600	4	1
1600—2000	1	3
2000—2400	—	4

Table 9 shows the range and number of occurrences of temperature changes.

Falls are relatively rare and heating can be boosted by the fan convector. Rises are much more common. Rises in the range 0-15°C initial temperature (5-20°C) can still be accommodated by bringing in air from outside to cool the inside temperature. Rises in the range 16-25°C initial temperature will not be able to do this as the external air will not be colder than the internal air. Rises in this range represent 20/76 = 26.37 per cent. Overheating may occur in these cases if the output of the walls is not adequately reduced by the room temperature's rising. This is possible on an average of 20/8 = 2.5 times a year.

5.3 Effect of small variations of outside temperature on room conditions

For steady conditions:
$q_w = hA\,(t_s — t_{ai})$ but $q_w = (C_f + C_v)(t_{ai} — t_{ao})$

If the outside temperature changes to $t_{ao}{}^1$, the room temperature will change to $t_{ai}{}^1$, and the heat output of the wall will change to $q^1{}_w$. This argument assumes that the water flow temperature variation with outside temperature is slower than the room temperature variation with outside temperature.

Table 8. Rises in morning.

Total period of rapid rise (hours)	Total rise °C	No of occurrences	No of equivalent 2-hour periods
4	10	3	3
3	8	6	2
3	7	8	2
3	6	3	2
		20	

We then have

$$q^1{}_w = hA\,(t_s — t_{ai}{}^1)$$
$$q^1{}_w = (C_f + C_v\ \ (t_s{}^1 — t^1{}_{ao})$$

i.e. $hA\,(t_s — t_{ai}) = (C_f + C_v)\ (t_{ai} — t_{ao})$
and $hA\,(t_s — t_{ai}{}^1) = (C_f + C_v)\ (t_{ai}{}^1 — t_{ao}{}^1)$

or
$$\frac{t_s — t_{ai}}{t_s — t_{ai}{}^1} = \frac{t_{ai} — t_{ao}}{t_{ai}{}^1 — t_{ao}{}^1}$$

Table 9. No of occurrences.

Initial temp range °C	Rise	Fall
0—5	6	0
6—10	14	1
11—15	36	2
16—20	19	5
21—25	1	3
	76	11

Table 10 shows that for variations of outside temperature in Spring up to 5K the room temperature rises by less than 5K.

The fan convector will provide 50 litre/sec of fresh air. So with fan on:— $C_v \rightarrow C_v + 0.05 \times 1300$W/K

$$q_w \quad = hA\,(t_s - t_{ai}) = (C_f + C_v)\,(t_{ai} - t_{ao} -)$$

$$q^1 w \quad = hA\,(t_s - t_{ai}{}^1)\ [C_f + (C_v + 65)]\ (t_{ai}{}^1 - t_{ao}{}^1)$$

$$\frac{t_s - t_{ai}{}^1}{t_s - t_{ai}} = \frac{(C_f + C_v + 65)}{(C_f + C_v)}\ \frac{(t_{ai}{}^1 - t_{ao}{}^1)}{(t_{ai} - t_{ao})}$$

$$C_f + C_v \quad = \frac{3500}{20}\,w = 175$$

$$\frac{t_s - t_{ai}{}^1}{t_s - t_{ai}} = \frac{240}{175}\ \frac{t_{ai}{}^1 - t_{ao}{}^1}{t_{ai} - t_{ao}} = 1.37\ \frac{(t_{ai}{}^1 - t_{ao}{}^1)}{(t_{ai} - t_{ao})}$$

Table 11 tabulates values of t_{ai}.

So, rises in outside temperature of 5K do not affect room temperatures by more than 3K, in the short term.

5.4 Balancing

For steady state, at design conditions:
Heat output from water for one side of the wall
$= ½w \times 4.187 \times 10^3\,(t_f — t_r)$
Heat output from wall $= hA\,(t_s — t_{ai})$

For pipe coils at 300mm spacing $\dfrac{t_s - t_{ai}}{½(t_f + t_r) - t_{ai}} = 0.35$

Therefore heat output from wall $= \dfrac{7}{40}\,hA\,[t_f + t_r - 2t_{ai}]$

For the following values $t_{ai}{}^1$ is given:

Table 10.

t_{ai}	t_s	t_{ao}	$\frac{t_s - t_{ai}}{t_s - t_{ao}}$	$t_{ao}^1 - t_{ao}$	$t_{ai} - t_{ai}^1$°C
20	30	10	½	5	2.5
20	30	15	2/3	5	3.0
20	25	10	1/3	5	1.6
20	25	15	½	5	2.5

Table 11.

t_{ai}	t_s	t_{ao}	t_{ao}^1	t_{ai}^1
20	30	10	15	21.3
20	30	15	20	22.7
20	25	10	15	20.9
20	25	15	20	22.1
20	35	5	10	20.5
20	35	10	15	21.5

Heat loss = $(C_v + C_f)(t_{ai} - t_{ao})$

In steady state:

$$½w \times 4.187 \times 10^3\,(t_f - t_r) = \frac{7}{40} hA\,[t_f + t_r - 2t_{ai}]$$
$$= (C_v + C_f)(t_{ai} - t_{ao}) \qquad (1)$$

In the incorrectly balanced condition:
Heat ourflow from water for one side of wall
$= ½w^1 \times 4.187 \times 10^3(t_f - t_r^1)$
Heat output from wall = $hA\,(t_s^1 - t_{ai}^1)$

$$\frac{t_s^1 - t_{ai}^1}{½\,[t_f + t_r^1 - 2t_{ai}^1]} = 0.35$$

Therefore heat output from wall $= \frac{7}{40} hA\,[t_f + t_r^1 - 2t_{ai}^1]$

Heat loss = $(C_v + C_f)(t_{ai}^1 - t_{ao})$

$$\text{And } ½w^1 \times 4.187 \times 10^3\,[t_f - t_r^1] = \frac{7}{40} hA\,[t_f + t_r^1 - 2t_{ai}^1]$$
$$= (C_v + C_f)(t_{ai}^1 - t_{ao})$$

Dividing (2) by (1)

$$\frac{w^1}{w}\frac{[t_f - t_r^1]}{[t_f - t_r]} = \frac{t_f + t_r^1 - 2t_{ai}^1}{t_f + t_r^1 - 2t_{ai}} = \frac{t_{ai}^1 - t_{ao}}{t_{ai} - t_{ao}}$$

For design conditions $t_f = 82$°C
$t_r = 66$°C
$t_{ai} = 18$°C
$t_{ao} = -1$°C

$$\frac{w^1}{9}\left[\frac{82 - t_r^1}{16}\right] = \frac{82 + t_r^1 - 2t_{ai}^1}{112} = \frac{t_{ai}^1 + 1}{19}$$

Using the second part of the equation

$$t_r^1 = \frac{1}{19}(150t_{ai}^1 - 1446)$$

substituting for t_r^1, and rearranging

$$t_w^1 = \frac{3004\,w^1/w - 16}{150\,w^1/w + 16}$$

Table 12.

w^1/w	t_{ai}^1°C
0.5	16.3
0.75	17.4
1	18
1.25	18.4
1.5	18.6

Table 12 shows the effect of incorrect balancing on the room temperature.

This shows that balancing the exact quantity of water passing through the coils is not critical with the range ±25 per cent.

6 Site measurements

A series of measurements were made in a typical two-person dwelling over four days in January 1978. This was in a period immediately prior to handover to the client, so no dwellings were tenanted.

Measurements were made in the living room and bedroom using thermocouples with sensors sellotaped to the walls in various places. Air temperatures were measured by mercury in glass thermometers placed centrally in the room.

Unfortunately the water temperature in the pipework had been raised above the design conditions by the mechanical subcontractor and were left at this temperature for some time. This was immediately corrected, but the heat build-up caused by this and the lack of air circulation generally in the dwellings, due to the security requirement of keeping the dwelling closed, meant that temperatures were in excess of the design conditions, and this has to a large extent affected the results.

6.1 Performance of the wall

The ratio of wall, pipe and room temperatures was calculated using average figures over the period.

$$\frac{\text{temperature on wall} - \text{temperature in room}}{\text{temperature of pipe} - \text{temperature in room}} \times 100$$
$$= \frac{40 - 24}{47 - 24} \times 100 = 69.5$$

This is very high compared with 32.5 from the analogue results. The room temperature remained more or less constant throughout the period. The wall surface temperature cooled only slowly both with and without ventilation. The mean water temperature was originally approximately 70°C, before being corrected, on the first day, to approximately 58°C, and on following days modulated according to the weather around 45°C. If it is assumed that the conditions in the room bear more resemblance to those produced by a mean water temperature of 70°C, i.e. that the lack of ventilation prevented the room achieving a steady state for a water temperature of 45°C, the ratio works out at 35.

6.2 Experimental results

Environmental temperature Living room
Table 13 tabulates measurements taken on site for the living room.

Table 13. Living room.

Time of reading	Ext. temp.	Internal air temp.	Mean rad. temp.	Heated wall surface temp.	Environ-mental temp.	N
5th 16.30	6.4	20.5	22.9	35.25	22.1	3
6th 10.50	7.0	21.5	22.25	33.8	21.75	0.5
15.40	5.25	21.0	22.4	33.35	21.93	1.7 Fan on
7th 12.20	6.5	20.0	21.2	34.4	20.8	0.6
14.30	7.5	20.0	24.2	35.6	22.8	6.0 Fan on + window half open
16.25	6.75	20.0	23.6	35.3	22.4	4.8 Fan on hot window shut
16.50	6.0	11.7	19.5	32.5	16.9	24.0 Windows wide open

$$\text{Using } t_{ei} - t_{ai} = \frac{(t_{ei} - t_{ao}) \times 14.1N}{375.6 + 14.1N}$$

$$(375.6 + 14.1N)(t_{ei} - t_{ai}) = 14.1N(t_{ei} - t_{ao})$$

$$375.6(t_{ei} - t_{ai}) = 14.1N(t_{ei} - t_{ao})$$

$$N = \frac{375.6}{14.1}\frac{(t_{ei} - t_{ai})}{t_{ai} - t_{ao}}$$

For 1.5 a/c in whole dwelling air flow = 0.049m^3/s = 176m^3/h
vol. of living room = 42.84
∴ 1.5 a/c in whole dwelling = 4 a/c in living room

$$\text{Using } t_{ei} - t_{ai} = \frac{(t_{ei} - t_{ao}) \times 13.3N}{362.9 + 13.3N}$$

$$N = \frac{362.9}{13.3}\frac{(t_{ei} - t_{ai})}{t_{ai} - t_{ao}}$$

For 1.5 a/c in whole dwelling air flow = 0.049m^3/s
vol. of bedroom = 41.66m^3
1.5 a/c in whole dwelling = 4 a/c in bedroom

Bedroom
Table 14 tabulates measurements taken on site in the bedroom.

Table 14. Bedroom.

Time of reading	Ext. temp. °C	Internal air temp. °C	Mean rad. temp. °C	Heated wall surface temp. °C	Environ-mental temp. °C	N h^{-1}
5th 16.30	6.3	27.2	29.3	39.2	28.6	1.7
6th 10.50	7.0	26.5	28.7	37.4	28.0	2.1
11.50	8.3	26.5	28.7	37.5	28.0	2.3
14.05	8.5	26.5	29.2	36.9	28.3	2.7
15.40	5.5	26.0	28.4	36.7	27.6	2.3 Fan on
18.25	5.0	24.0	28.2	36.8	26.8	6.9
7th 10.30	6.0	21.0	27.0	38.9	25.0	7.3 Window open
12.20	6.5	21.0	26.8	37.8	24.9	7.2 ,, ,,
14.30	7.5	24.0	27.3	37.7	26.2	3.6 Fan on
16.15	6.75	26.5	29.6	38.2	28.6	2.9 ,, ,,
17.15	6.0	14.0	25.8	36.2	21.9	26.9 Windows wide open

7 Costs

The two alternatives compared at the cost plan stage of the design were a radiator system and the pipe coil wall heating. Heat was to be provided from a central boilerhouse in both cases, and the distribution pipework was identical. The difference in cost for these two alternatives is therefore in the installation costs in each dwelling, which are compared below (at 1970 prices).

7.1 Pipe coil system

Costs per 10 dwellings = £600 (see Table 15).

7.2 Radiator scheme

Cost per 10 dwellings = £1300
Therefore saving with pipe coil system = £130 — £60 = £70 per dwelling.

Mechanical ventilation was required in both cases and the cost of this, based on the provision of a small fan in each room requiring mechanical ventilation, was £95 per dwelling. This included provision for sound-proofing, as it would not be acceptable for noise reasons merely to install a simple system such as Ventaxia in a window. In order to simplify the maintenance problems, the number of fans was reduced to one per dwelling, and although this necessitated more ductwork, it enabled a heater battery to be incorporated for no extra cost. The total extra cost of the pipe coil

Table 15. Costs—pipe coil system.

Cost per riser (10 dwellings)	£
Coils supply	120
Builders' work (see summary below)	400
Pipe connections to risers	77
	597
Builders' work (total 346 dwellings)	
Limestone aggregate : extra over shingle	3500
Extra reinforcement	6500
Heating coil fixing costs	3000
Work in connection (pockets for pipe connections)	500
	£13500
	(approx. £40 per dwelling)

and mechanical ventilation system, over a conventional radiator scheme without mechanical ventilation, was therefore only £25 per dwelling.

8 Conclusions

This paper outlines a method of designing radiant wall heating by steel pipe coils embedded in concrete cross walls. It provides details of outputs to be expected for particular pipe spacings and temperatures. Anticipated room conditions and control problems are investigated.

A preliminary site test shows that the temperatures predicted in the walls and the rooms are being reached. However no detailed investigation on site has yet been done. At the time of writing tenants have been in occupation for six months, including the cold spells of the winter. Dwelling temperatures seem to be higher than predicted and an experiment in reducing the flow temperature has produced no complaints so far.

While this type of heating is limited in that it can only be installed in concrete walls, the designers suggest that it is a more satisfacory form of heating than floor or ceiling warming, and that the savings in capital costs would justify its use more often. it is also possible that the flow temperature reduction will produce additional fuel savings.

Appendix

Thermal capacity of building A block only

The calculations in this section are very approximate and do not pretend to be a detailed investigation of the non steady state conditions.

Each wall has average area = $6.86 \times 2.59 \times 8 = 142.0\text{m}^2$
Walls are 178mm thick
Volume of wall = $142 \times 0.178\text{m}^3$
= 25.2m^3
Weight of concrete say 2400kg/m^3
Therefore weight of wall = $25.2 \times 2400 = 6.06 \times 10^4\text{kg}$
Specific heat of heavy building = 840 J/kgK
Thermal capacity of 36 walls = $36 \times 840 \times 6.06 \times 10^4\text{J/K}$
= $1.833 \times 10^9\text{J/K}$
For Spring conditions say water flow = 65°C
water return = 60°C
concrete av temp = 35°C
Design Winter conditions waterflow = 82°C
water return = 65°C
concrete av temp = 41°C

For Outside temperature drops

Centre of wall near pipe has to heat up from 65°C to 82°C = 17K
Wall surface has to heat up from 35°C to 41°C = 6K
Therefore take average temp rise as 11.5K
Therefore concrete requires $1.833 \times 10^9 \times 11.5$J to heat up
= 2.11×10^{10}J
A block design heat loss = 2110kW

$$\text{Building will take } \frac{2.11 \times 10^{10}}{2110000 \times 3600} = 2.8 \text{ hours to heat up}$$

In these conditions, assume all buildings will demand heat and there is no spare capacity from boilers
Water content of system = 9×10^4kg
This must be heated from 60°C to 82°C at the rate of 2110kW

$$\text{Water will take } \frac{9 \times 10^4 \times 22 \times 4.187 \times 10^3}{2110 \times 10^3 \times 3600} \text{ hours to heat up}$$

$$= 1.1\text{h}$$

If this total $2.7 + 1.1 \simeq 4$ hours is critical, then an advance pulse of water must be sent round the building four hours before a cold spell is due, so that the building is already warming up. The pulse can be provided with more boiler plant available say ½ of total, so pulse will be available after

$$\frac{9 \times 10^4 \times 22 \times 4.187 \times 10^3}{0.5 \times 5.86 \times 10^6 \times 3600} = 0.79 \text{ say } \tfrac{3}{4}\text{ h}$$

For Outside temperature rises

Heat output of wall = $hA\,(t_s - t_{ai})$

$$\frac{t_s - t_{ai}}{t_p - t_{ai}} = 0.33 \text{ for the particular walls} \qquad (1)$$

Heat loss from dwelling = $(C_v + C_f)\,(t_{ai} - t_{ao})$
Heat loss from the dwelling = heat output of wall

$$\therefore (C_v + C_f)\,(t_{ai} - t_{ao}) = hA\,(t_s - t_{ai}) \qquad (2)$$

When temperature rises $t_{ao} \rightarrow t_{ao}^1$ $t_s \rightarrow t_s^1$ $t_p \rightarrow t_p^1$
When steady state is achieved t_{ai} is constant.

$$\therefore (C_v + C_f)\,(t_{ai} - t_{ao}^1) = hA\,(t_s^1 - t_{ai}) \qquad (3)$$

dividing 3 by 2

$$\frac{t_{ai} - t_{ao}^1}{t_{ai} - t_{ao}} = \frac{t_s^1 - t_{ai}}{t_s - t_{ai}}$$

$$\text{or re-arranging} \quad t_s - t_s^1 = \frac{(t_s - t_{ai})(t_{ao}^1 - t_{ao})}{(t_{ai} - t_{ao})}$$

$t_p - t_{ai} = 0.33\,(t_s - t_{ai})$

$$\text{so similarly} \quad t_p - t_p^1 = \frac{(t_p - t_{ai})(t_{ao}^1 - t_{ao})}{(t_{ai} - t_{ao})}$$

For design conditions $t_{ao} = -1°\text{C}$ $t_{ai} = 18.3°\text{C}$ $t_p = 74°\text{C}$

$$\frac{t_s - t_{ai}}{t_p - t_{ai}} = 0.33$$

$$t_s = 36.7°C$$

$$36.7 - t_s^1 = \frac{36.7 - 18.3\,(t_{ao}^1 + 1)}{18.3 + 1}$$

$$= 0.95\,(t_{ao}^1 + 1)$$

$$t_s^1 = 36.7 - 0.95\,(t_{ao}^1 + 1)$$

$$t_p^1 = t_p - \frac{(t_p - t_{ai})\,(t_{ao}^1 - t_{ao})}{t_{ai} - t_{ao}}$$

$$= 74 - \frac{74 - 18.3}{18.3 + 1}(t_{ao}^1 + 1)$$

$$= 74 - 2.89\,(t_{ao}^1 + 1)$$

$$t_{ao}^1 = 4°C$$

$$t_s^1 = 32°C$$

$$t_p^1 = 60°C$$

Centre of wall has to cool from t_p to t_p^1 = 74°C to 60°C = 14K
Surface of wall has to cool from t_s to t_s^1 = 36.7°C to 32°C = 4.7K
∴ Average cooling = 9.35°C
Thermal capacity of wall in one dwelling = $tA\rho s$
A = area of wall = 17.77m^2
t = thickness of wall = 0.178m
ρ = density of concrete = 2400kg/m^3
s = specific heat of concrete = 840J/kgK
∴ thermal capacity = 6.38×10^6J/K
Heat to be lost = $6.38 \times 10^6 \times 9.35 = 60 \times 10^6$J
Rate of losing heat on one side of the wall = $hA\,(t_w - t_a)$
where h = surface heat transfer coefficient say 8W/m^{2}K
taking t_w as the average value
Rate of losing heat for both sides of wall
$= 2 \times 8 \times 17.77(34.35 - 18.3)$
$= 4.56 \times 10^3$J/s

∴ Wall takes $\frac{60 \times 10^6}{4.56 \times 10^3}$ secs to cool down

So a 5K rise in external temp results in a 3.65 hour time lag in the wall achieving the steady state condition for the new external temperature.

References

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